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Mechanical Response of a Micro Silicon Membrane: Model Validation by Finite Element Method

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Abstract

In the piezoresistive pressure sensor, we need to study the stress repartition on the Silicon membrane surface. This study is very important because it allows us to determinate where the stress is maximal and to place there the four piezoresistors, which provide a maximal sensitivity to the pressure [2]. Because of the anisotropy of the Silicon, we have to study the variation of its mechanical parameters (Young Modulus and Poisson Coefficient) and piezoresistive (piezoresistive coefficients) parameters in the different crystallographic directions [4]. These studies will allow us to optimize the sensor characteristics (Sensitivity, Linearity …).

In this paper, we propose to establish a mathematical model that describes the mechanical behavior of single crystal silicon micro membrane. The proposed model is based on the theory of thin plates and shell. It describes the deflection of the membrane and the stresses induced in by the effect of a homogeneous differential pressure applied to this membrane. The resolution of this model allows us to calculate the displacement of the membrane (his deflection) and to know the stress distribution as a function of the applied pressure. Finally, results are compared with those obtained by the numerical finite elements method.

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Keywords: Piezoresistivity, Poisson Coefficient, Silicon, Sensitivity, Sensor, Stress, Theory of plates, Young Modulus.

1. Introduction

Since thirty years, pressure sensors are constantly evaluating and dominate the sensors market by occupying the most important place in modern industrial applications [5]. This type of sensors has benefited from the advent of microelectronics technology to move from the simple gauges made one by one to micro sensors made in series with compact size and low coast.

In this paper, we will study a piezoresistive micro pressure sensor which the test-body is a thin Silicon membrane perfectly clamped on its boards. We will model, at first, its mechanical behavior based on the theory of elasticity in cubic crystals and the theory of thin plates to determine the flex of the membrane under the

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pressure applied and the stress repartition on the membrane surface.

2. Piezoresistivity in Semiconductors

Semiconductors are characterized by a very important piezoresistive coefficient (about 20 times largest than those in metals and alloys). Hence, scientists give a special interest to semiconductor materials and especially to Silicon.

Smith [6] has experimentally demonstrated that (in Silicon and Geranium) the relative variation of resistivity is proportional to the applied stress. Therefore, he established the following equation:

$$\frac{\partial\rho}{\rho} = \sum\pi_{ij}.\sigma_{kl} \tag{1}$$

σ_{kl} is the piezoresistive coefficient. It depends on the crystallographic properties of the semiconductor, its doping concentration and its temperature. Equation 1 shows that the sensor sensitivity will be more important when the piezoresistive coefficient and strain are going greater. We, therefore, will try to maximize this terms.

3. Analytical model based on thin plates and shell theory

We consider a square Silicon membrane (Fig. 1) situated on the (Oxy) plan. Its dimensions are "a" in each side and "h" in thickness.

To modeling the mechanical behavior of this membrane, we will apply the theory of thin plates and the theory of elasticity. However, some conditions should be taken in consideration:

- The membrane is perfectly clamped on its four boards.
- The applied pressure should not exceed the elasticity limit of the considered material.
- The membrane is assimilated to a thin plate (h << a).
- There are no external forces applied to the membrane.

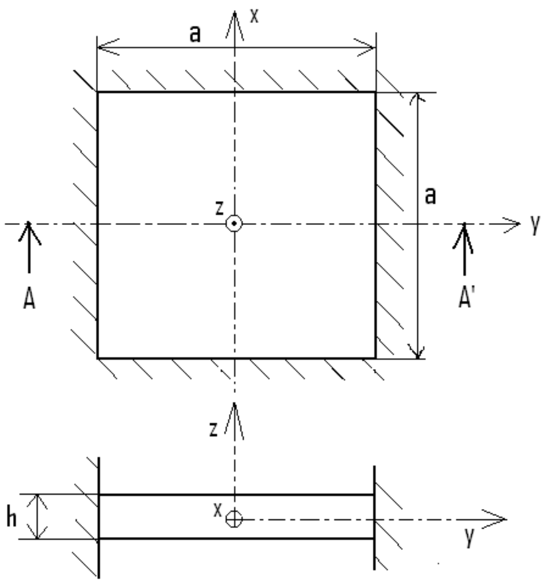

Fig. 1. Top view and cross section view of the membrane.

Under a constant and uniform pressure parallely applied to (Oz) direction, the membrane will temporally flex. Its movement is described by a PDE (partial differential equation), called Lagrange equation:

$$\frac{\partial^4 W(x,y)}{\partial x^4} + 2\alpha_{Si}\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} = \frac{P}{D} \tag{2}$$

$$\alpha_{Si} = \vartheta + \frac{2G(1-\vartheta^2)}{E} \tag{3}$$

$$D = \frac{Eh^3}{12(1-\vartheta^2)} \tag{4}$$

α_{si} and D are respectively given by equation 3 and 4. W(x,y) is the flex of the membrane calculated on the (Oz) direction in each (x,y) point.

Other part, the theory of elasticity provides relationship between normal and shear stress and the flex W(x,y). This relationship of the normal stress and shear stress are given respectively by equations 5 and 6.

$$\sigma_x = -\frac{h}{2}\frac{E}{1-\vartheta^2}\left(\frac{\partial^2 W(x,y)}{\partial x^2} + \vartheta\frac{\partial^2 W(x,y)}{\partial y^2}\right) \tag{5}$$

$$\tau_{xy} = -hG\left(\frac{\partial^2 W(x,y)}{\sigma_x \sigma_y}\right) \tag{6}$$

4. Resolution of the analytical model

Perfect clamping of the membrane imposes some limits conditions:

$$\begin{cases} W\left(x = \mp\frac{a}{2}, y\right) = 0 \\ W\left(x, y = \mp\frac{a}{2}\right) = 0 \end{cases} \tag{7}$$

$$\begin{cases} \frac{\partial W}{\partial x}\left(x = \mp\frac{a}{2}, y\right) = 0 \\ \frac{\partial W}{\partial y}\left(x, y = \mp\frac{a}{2}\right) = 0 \end{cases} \tag{8}$$

By solving the system of equation formed by equations (2), (5), (6), (7) and (8) we can get the flex W(x,y) at any point of the membrane [3]. Nevertheless, the exact solution of this analytical system is not known. For this, we will use the approximate method of Galerkin. The approximate solution is given by equation 9.

$$W(x,y) = \frac{P}{16D}\frac{a^4}{h^3}\left[1-\left(\frac{2x}{a}\right)^2\right]^2\left[1-\left(\frac{2y}{a}\right)^2\right]^2\left\{\sum_{i=0}^{n}\sum_{j=0}^{n} k_{ij}\left(\frac{2x}{a}\right)^i\left(\frac{2y}{a}\right)^j\right\} \tag{9}$$

P is the applied pressure, i and j are even integers and k_{ij} are coefficients depending on the anisotropy factor α_{Si}.

Once the k_{ij} coefficients calculated, we can plot the normalized flex (Figure 2) of the membrane that is maximal at its center and null in the boards because of the perfect clamping there.

The stress repartition is also given in figure 3 and 4. These figures illustrate, respectively, the repartition of the normal and shear stress in the surface of the membrane. We remark that stresses are very important on the boards of the membrane. Hence, the best emplacement of gauges will be there.

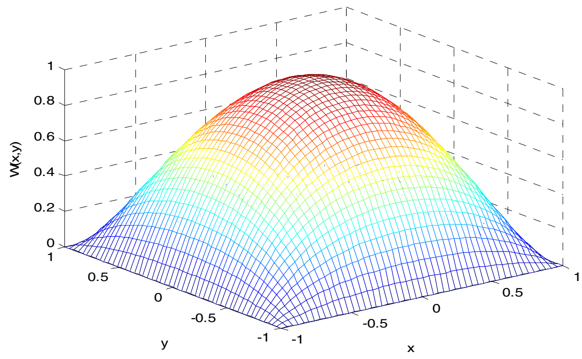

Fig. 2. Flex of the membrane (3D).

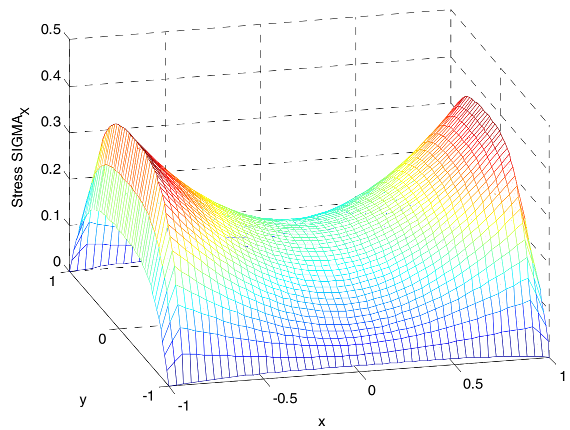

Fig. 3. Repartition of the normal stress σ_x.

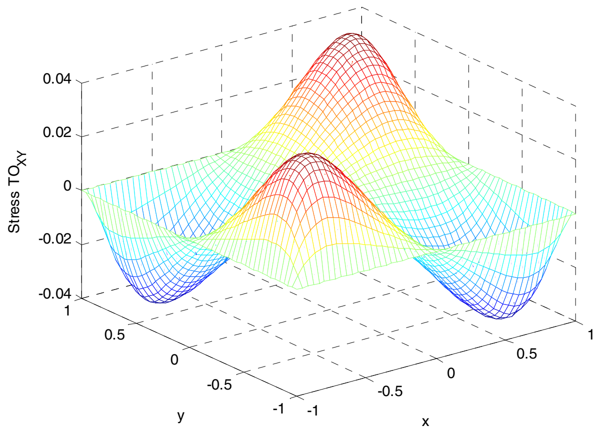

Fig. 4. Repartition of the shear stress τ_{xy}.

5. Comparison and validation of results by finite elements method

Then, in order to check the conformity of the semi-analytical method of Galerkin and validate the results obtained, we propose to solve the same model with another method, the finite element method which is known for its very good accuracy. The obtained results are given in Figure 5, 6 and 7.

Comparison of results will allow us to know the benefits of semi-analytical method compared to the numerical one, but also to know the disadvantages and limitations of Galerkin method.

After comparison, we remark that there is a convergence between the results obtained by both methods; witch leads us to adopt the semi-analytical method which is much less expensive in terms of computation time.

However, the semi-analytical model does not support multi-layered structures where the mechanical properties of those structures are not homogeneous [1].

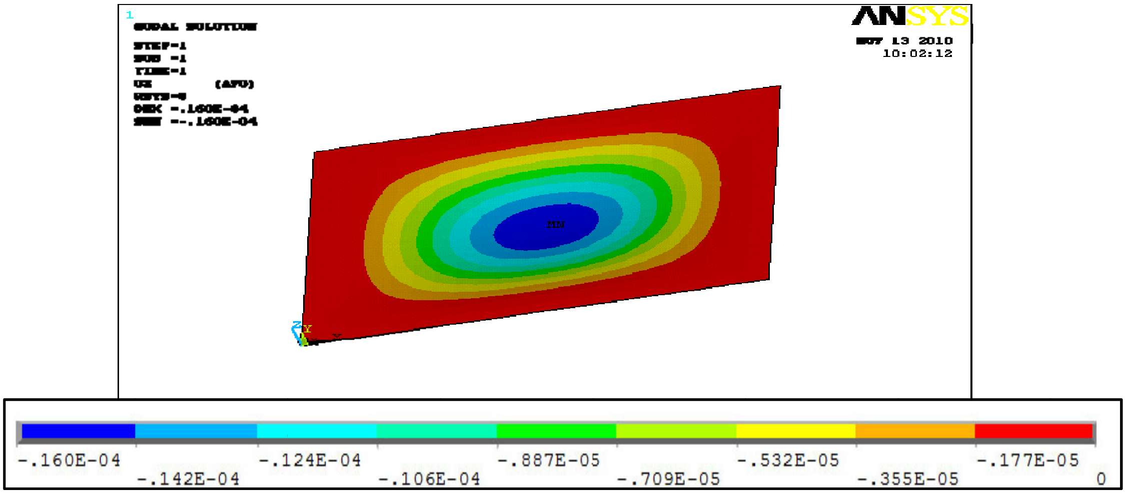

Fig. 5. Flex of the membrane obtained by FEM.

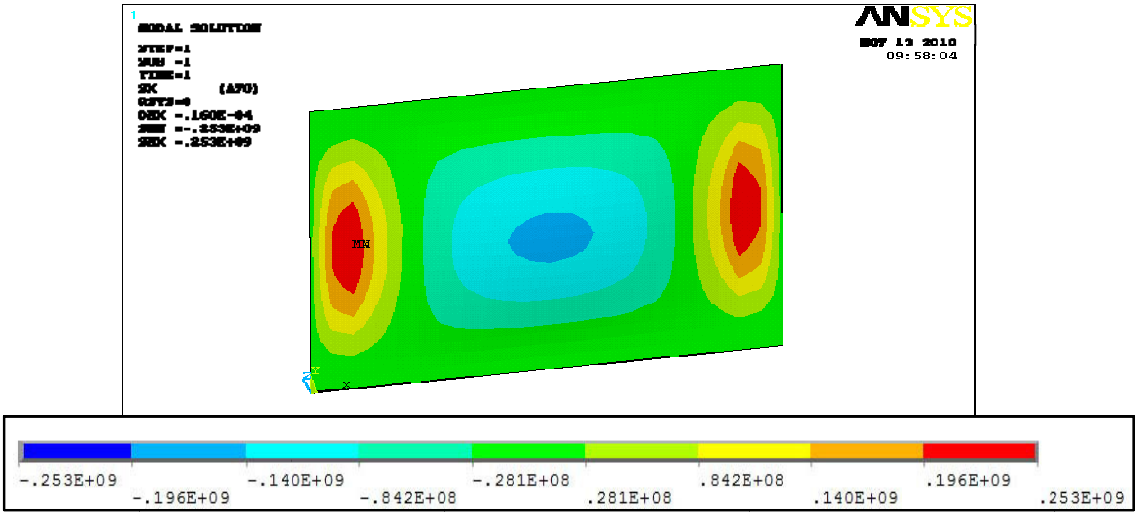

Fig. 6. Repartition of the normal stress σ_x obtained by FEM.

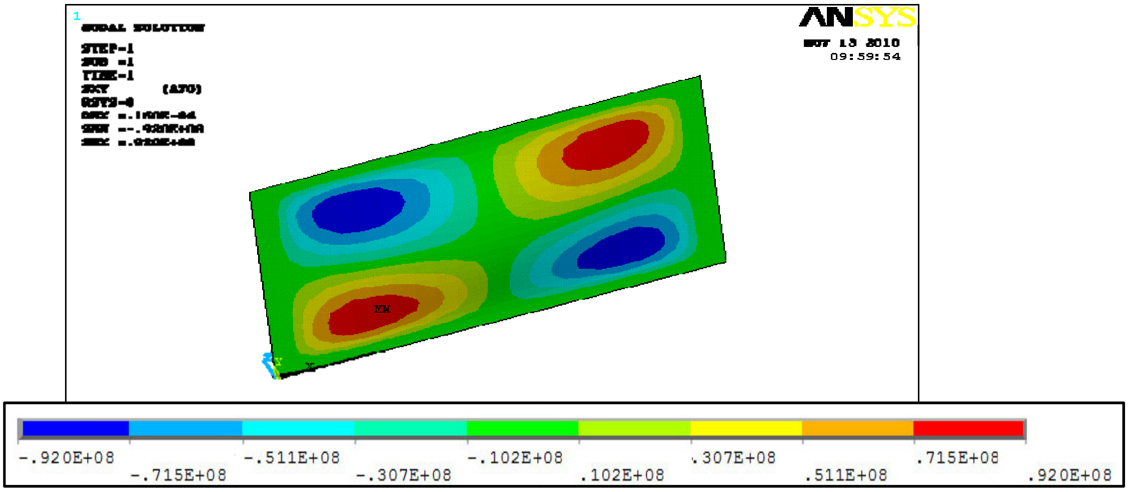

Fig. 7. Repartition of the shear stress τ_{xy} obtained by FEM.

6. Conclusion

In conclusion, this paper has demonstrated that it is possible to model, with good precision, the mechanical behavior of the piezoresistive micro pressure sensor by adopting a simple approach based on the theory of thin plates and shell. The resolution of this model gives results which are in good accuracy with those obtained by numerical method. The simplicity of the analytical model is traduced by a very low time of calculation in comparison with the numerical model. Nevertheless, the analytical model can't take in consideration residual stress caused by thermo mechanical effect.

Acknowledgements

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